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Noisy group testing: achievable rates

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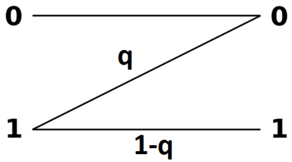
Overview

- Problem Setup and Motivation
- Dilution Channel Noise (DCN)
- NCOMP Algorithm
- Achievability Bound for NCOMP
- Experiments
- Further work

Problem Setup and Motivation

- n: total number of items;
- $d = \Theta(n^{\theta})$: number of defective items;
- $\mathcal{D}$: set of defective items;
- $T = \beta d \log n$: number of tests sufficient for vanishing P_e;
- $\boldsymbol{\sigma}$: Parameter of interest (Boolean vec.) defectives (1) & non-defectives (0);
- $\hat{\boldsymbol{\sigma}}$: Estimate (Boolean vec.) defectives (1) & non-defectives (0);
- M: $T \times n$ design matrix ($m_{(i,j)} \sim$ Bernoulli$(\frac{\alpha}{d})$);
- $q \in (0, 1)$: noise parameter in Dilution Channel Noise.

Dilution Channel Noise (DCN) [1]



$$\mathbf{y}_i = \bigvee_{j=1}^{n} \mathcal{Z}(M_{(i,j)} \cdot \mathcal{I}_j)$$

Figure: DCN (left); test resuls in DCN (right)

- Each item initially included in a given test pool is diluted with prob. q ;
- From [1], $T = \mathcal{O}(\frac{d \log n}{(1-q)^2})$ guarantees asymptotically vanishing P_e;
- Unknown achievable bounds for computationally feasible algorithms (e.g. NCOMP, NDD).

NCOMP Algorithm [2]

- $\mathcal{T}_i$: indices of tests where item i appears;
- $\mathcal{S}_i$: indices of positive tests where item i appears;
- **NCOMP**: $i \in \mathcal{D} \Leftrightarrow |\mathcal{S}_i| \geq |\mathcal{T}_i| \cdot (1 - q \cdot (1 + \Delta))$;
- Δ: design parameter (Noise robustness).

NCOMP Algorithm

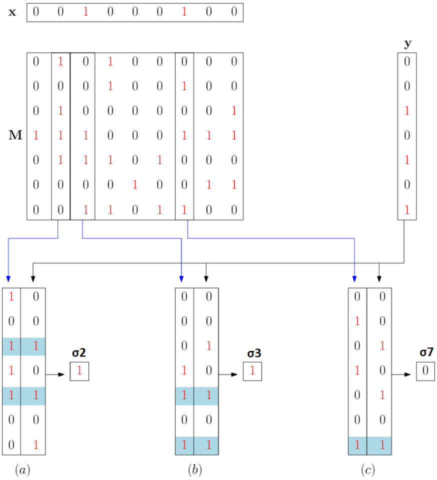


Figure: NCOMP - Stylised example

Achievability Bound for NCOMP

Theorem

For noisy group testing under DCN with parameter $q \in (0,1)$, $d = \Theta(n^{\theta})$, $\theta \in (0,1)$, NCOMP achieving $P_e \to 0$ requires no more than

$$\frac{(1+\epsilon)d \log n}{q^2\alpha(1-e^2)((1+\Delta)-(1-\frac{\alpha(1-q)}{d})^{d-1})^2}$$

tests, given that

$$1+\Delta := \frac{(1-\frac{\alpha(1-q)}{d})^{d-1}+\frac{1}{q}(1-\frac{\alpha(1-q)}{d})^{d}}{2}$$

for Bernoulli($\frac{\alpha}{d}$) designs and $\epsilon > 0$.

Achievability Bound for NCOMP

- Results obtained for d, q known;
- Generalisations are possible by assuming only bounds are known;
- The results are order-wise tight, but analytical derivations lead to sub-optimal constants;
- Achievable bounds remain somewhat distant from (unknown) thresholds, empirically;
- Large improvements are obtained by assuming upper bounds on θ (sparsity level);
- Numerical solvers are however needed.

Experiments

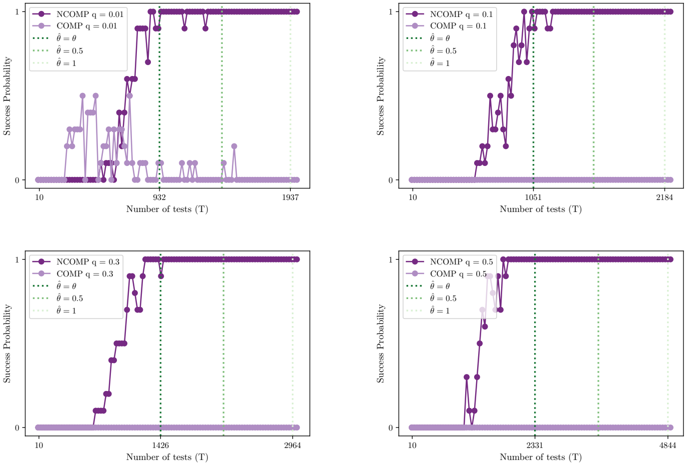


Figure: Results of numerical simulations of COMP and NCOMP on dilution channel noise for different noise levels q; $n = 10000$, $\theta = 0.15$.

Experiments

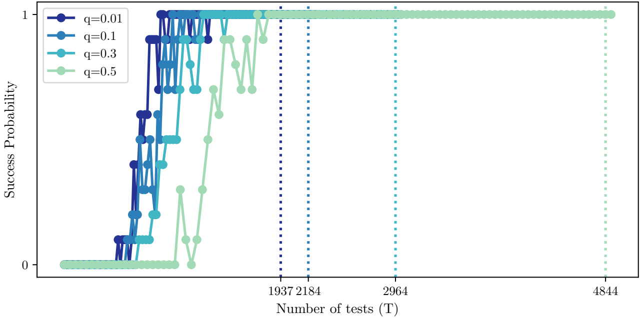


Figure: NCOMP with Dilution noise for different noise levels q; $n = 10000$, $\theta = 0.15$. Dotted line is the achievability bound from Theorem 1, corresponding to $\hat{\theta} = 1$.

Further Work

- Analyse improvements on Achievable bound from sparsity constraints;
- Derive Converse bound (Sharp threshold) on NCOMP;
- Complete a similar analysis for more refined algorithms (NDD);
- Find tailored algorithms for group testing in DCN.

References

[1] G. K. Atia and V. Saligrama, "Boolean compressed sensing and noisy group testing," *IEEE Transactions on Information Theory*, vol. 58, no. 3, pp. 1880–1901, 2012.

[2] J. Scarlett and O. Johnson, "Noisy non-adaptive group testing: A (near-) definite defectives approach," *IEEE Transactions on Information Theory*, 2020.